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Indian Statistical Institute
B.Math.(Hons.) II Year
First Semester Mid Semester Examination, 2005-2006
Algebra III

Time: 3 hrs | Date:14-09-05 | Total Marks : 50

Attempt all questions

1. Examine each statement below, state whether they are true or false and justify your answer.

   (a) $Q(\sqrt{2})$ and $Q(\sqrt{3})$ an isomorphic as Q-vector spaces but not as fields.

   (b) $Q(\sqrt{3}+\sqrt{5}) = Q(\sqrt{3}, \sqrt{5})$.

   (c) $[Q(\sqrt{3+2\sqrt{2}}) : Q] = 4$.

   (d) A finite abelian group can always be given the structure of a Q-module.

   ($5 \times 4 = 20$ marks)

2. Let F be a finite field

   (a) Prove that F^* is a cyclic group under multiplication.

   (b) Given any positive integer n, prove that there exists an irreducible polynomial over F of degree n. ($6 + 6 = 12$ marks)

3. (a) Find a basis for the $\mathbb{Z}$- submodule M of $\mathbb{Z}^3$ defined by

$$M = \{(x_1,\ x_2,\ x_3) \in \mathbb{Z}^3 / x_1 + 2x_2 + 3x_3 = 0,\ x_1 + 4x_2 + 9x_3 = 0\}$$

(b) Let $I = (2,\ x) \subseteq \mathbb{Z}[x]$ be the ideal considered as a $\mathbb{Z}[x]$-submodule of $\mathbb{Z}[x]$. Show that I is not a direct sum of cyclic $\mathbb{Z}[x]$-modules.

(6+6 = 12 marks)

4. Let F be any field and $f(x) \in F[x]$ be an irreducible polynomial of degree 5. Let K/F be an extension with $[K : F] = 2$. Prove that $f(x)$ remains irreducible in $K[x]$. (6 marks)